Simplification of a force and couple (replace to force and couple)

- **If the system in 3D, represent each force as a Cartesian vector before summing the forces.**

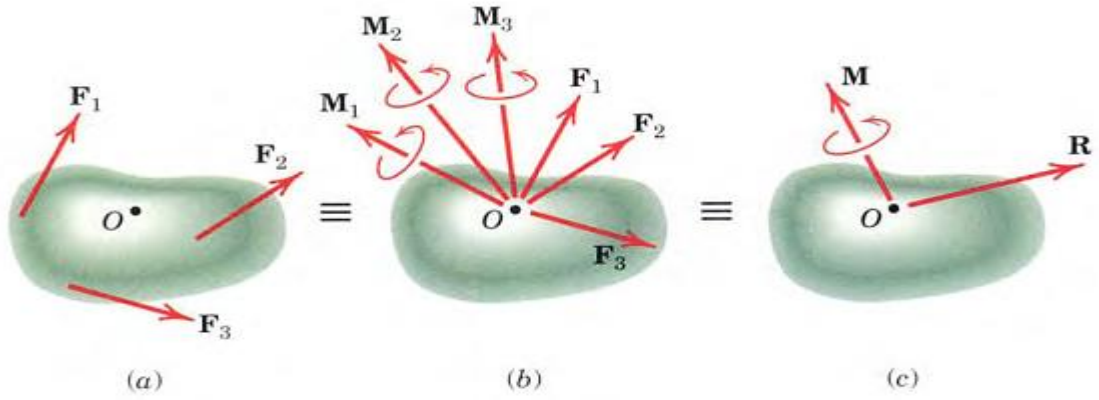
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F}$$

- **Use the vector cross product to determine the moment of each force about point 0 , Here the position vectors extend from a to any point on the line of action of each force.**

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$

- **Use the dot product to find the angle between them.**

$$\cos\theta = \frac{R.M}{|R||M|}$$



Replace to a wrench (magnitude force and parallel moment)

- **Make the same previous steps.**
- **Find the magnitude of the resultant force.**
- **Parallel moment can be found as,**

$$M_{||} = \frac{\vec{R}.\vec{M}}{|R|}$$

- **Pitch (P) found by the equation,**

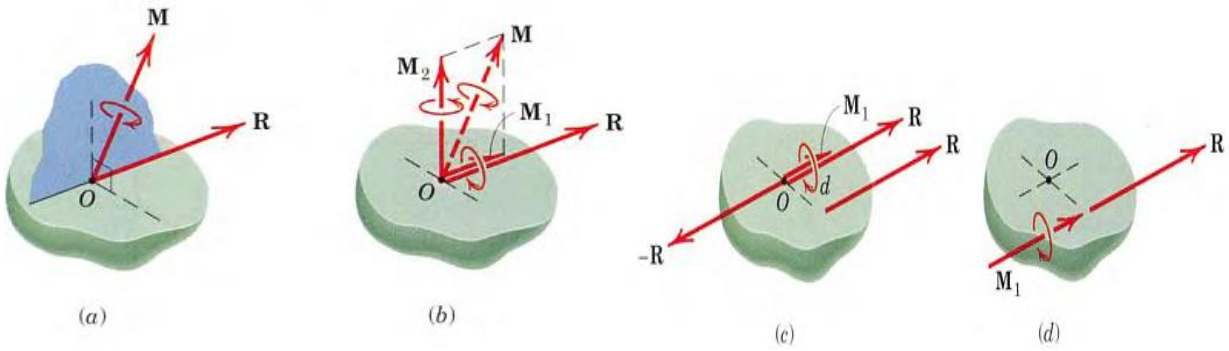
$$P = \frac{M_{||}}{|R|} = \frac{\vec{R}.\vec{M}}{|R|^2}$$

- **Axis $\vec{r}$ is**

$$\vec{r} = \frac{\vec{R} \times \vec{M}}{|R|^2} + \lambda \vec{R} \qquad \textbf{OR} \qquad \vec{M}_{\perp} = \vec{r} \times \vec{R} = \begin{vmatrix} i & j & k \\ x & y & z \\ R_x & R_y & R_z \end{vmatrix}$$

In part *b* of the figure, **M** is resolved into components $\mathbf{M}_1$ along the direction of **R** and $\mathbf{M}_2$ normal to **R**. In part *c* of the figure, the couple

$\mathbf{M}_2$ is replaced by its equivalent of two forces **R** and $-\mathbf{R}$ separated by a distance $d = M_2/R$ with $-\mathbf{R}$ applied at *O* to cancel the original **R**. This step leaves the resultant **R**, which acts along a new and unique line of action, and the parallel couple $\mathbf{M}_1$, which is a free vector, as shown in part *d* of the figure. Thus, the resultants of the original general force system have been transformed into a wrench (positive in this illustration) with its unique axis defined by the new position of **R**.



Examples

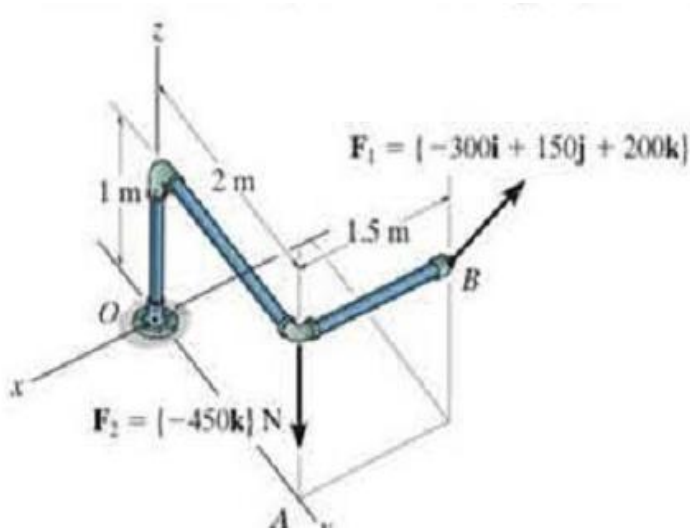
1- Replace the loading system by

A. An equivalent resultant force and couple then find the angle between them.

B. A wrench specify the pitch and the axis

Solution

A.



$$\vec{F}_1 = -300\vec{i} + 150\vec{j} + 200\vec{k}$$

$$\vec{F}_2 = -450\vec{k}$$

$$\vec{R} = -300\vec{i} + 150\vec{j} - 250\vec{k}$$

$$\vec{M}_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$= \begin{vmatrix} i & j & k \\ -1.5 & 2 & 1 \\ -300 & 150 & 200 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 0 & 0 & -450 \end{vmatrix}$$

$$= (250\vec{i} + 375\vec{k}) + (-900\vec{i})$$

$$= -650\vec{i} + 375\vec{k}$$

$$|R| = 50\sqrt{70}$$

$$|M_0 = 25\sqrt{901}|$$

$$\cos\theta = \frac{\vec{R}.\vec{M}_0}{|R||M|} = \frac{101250}{313921.7657} = 0.323$$

$$\theta = 71.18$$

B.

$$|R| = 50\sqrt{70}$$

$$|M_0 = 25\sqrt{901}|$$

$$M_{||} = \frac{\vec{R}.\vec{M}}{|R|} = \frac{101250}{50\sqrt{70}} = 242.03$$

$$P=\frac{M_{||}}{|R|}=\frac{\vec{R}.\vec{M}}{|R|^2}=\frac{242.03}{50\sqrt{70}}=0.58$$

$$\vec{r}=\frac{\vec{R}\times\vec{M}}{|R|^2}+\lambda\vec{R}=\frac{1}{(50\sqrt{70})^2}\begin{vmatrix} i & j & k \\ -300 & 150 & -250 \\ -650 & 0 & 375 \end{vmatrix}+\lambda(-300\vec{i}+150\vec{j}-250\vec{k})$$

$$=\frac{9}{28}\vec{i}+\frac{11}{7}\vec{j}+\frac{39}{70}\vec{k}+\lambda(-300\vec{i}+150\vec{j}-250\vec{k})$$

$$x\vec{i}+y\vec{j}+z\vec{k}=(\frac{9}{28}-300\lambda)\vec{i}+(\frac{11}{7}+150\lambda)\vec{j}+(\frac{39}{70}-250\lambda)\vec{k}$$

Or

$$\vec{M}_{||}=P\vec{R}$$

$$\vec{M}_{\perp}=\vec{M}_0-\vec{M}_{||}=\vec{M}_0-P\vec{R}$$

$$=650\vec{i}+375\vec{k}-0.58(-300\vec{i}+150\vec{j}-250\vec{k})$$

$$=824\vec{i}+150\vec{j}+520\vec{k}$$

$$\vec{M}_{\perp}=\vec{r}\times\vec{R}=\begin{vmatrix} i & j & k \\ x & y & z \\ R_x & R_y & R_z \end{vmatrix}$$

$$824\vec{i}+150\vec{j}+520\vec{k}=\begin{vmatrix} i & j & k \\ x & y & z \\ -300 & 150 & -250 \end{vmatrix}=(-250y-150z)\vec{i}-(-250x+300z)\vec{j}+(150x+300y)\vec{k}$$

2- Replace the loading system by

A. An equivalent resultant force and couple then find the angle between them.

B. A wrench specify the pitch and the axis

Solution

A.

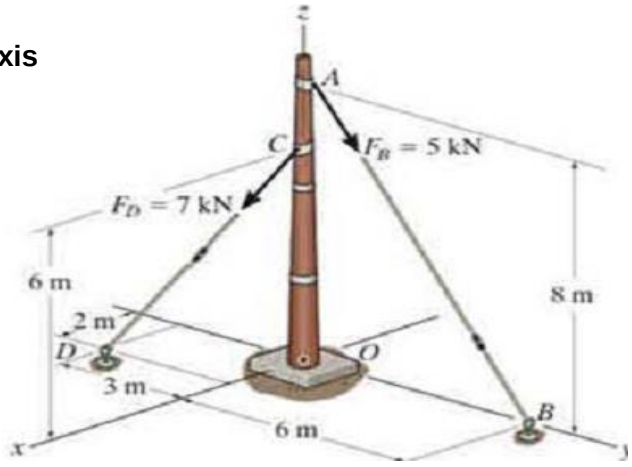
$$\vec{F}_B=|F_B|\frac{\overrightarrow{AB}}{|AB|}$$

$$A=(0,0,8)$$

$$B=(0,6,0)$$

$$C=(0,0,6)$$

$$D=(0,2,-3)$$



$$\frac{\overrightarrow{AB}}{|AB|}=\frac{B-A}{|AB|}=\frac{(0,6,-8)}{\sqrt{6^2+8^2}}=\frac{(0,6,-8)}{10}$$

$$\vec{F}_B=5*\frac{1}{10}(0,6,-8)=3\vec{j}-4\vec{k}$$

$$\vec{F}_D=|F_D|\frac{\overrightarrow{CD}}{|CD|}$$

$$\frac{\overrightarrow{CD}}{|CD|}=\frac{D-C}{|CD|}=\frac{(2,-3,-6)}{7}$$

$$\vec{F}_D=7*\frac{1}{7}(2,-3,-6)=2\vec{i}-3\vec{j}-6\vec{k}$$

$$\vec{R}=2\vec{i}-10\vec{k}$$

$$\vec{M}_0=\vec{r}_1\times\vec{F}_1+\vec{r}_2\times\vec{F}_2$$

$$=\begin{vmatrix} i & j & k \\ 0 & 0 & 8 \\ 0 & 3 & -4 \end{vmatrix}+\begin{vmatrix} i & j & k \\ 0 & 0 & 6 \\ 2 & -3 & -6 \end{vmatrix}$$

$$=-24\vec{i}+18\vec{i}=-6\vec{i}$$

$$|R|=2\sqrt{26}$$

$$|M_0|=6$$

$$\cos\theta=\frac{\vec{R}.\vec{M}_0}{|R||M|}=\frac{12}{12\sqrt{26}}=\frac{\sqrt{26}}{26}$$

$$\theta=78.69$$

B.

$$|R|=2\sqrt{26}$$

$$|M_0|=6$$

$$M_{||}=\frac{\vec{R}.\vec{M}}{|R|}=\frac{12}{2\sqrt{26}}=\frac{6}{\sqrt{26}}$$

$$P=\frac{M_{||}}{|R|}=\frac{\vec{R}.\vec{M}}{|R|^2}=\frac{3}{\sqrt{26}}$$

$$\vec{r}=\frac{\vec{R}\times\vec{M}}{|R|^2}+\lambda\vec{R}=$$

$$\frac{1}{(2\sqrt{26})^2}\begin{vmatrix} i & j & k \\ 2 & 0 & -10 \\ 6 & 0 & 0 \end{vmatrix}+\lambda(2\vec{i}-10\vec{k})$$

$$=\frac{5}{13}\vec{j}+\lambda(2\vec{i}-10\vec{k})=$$

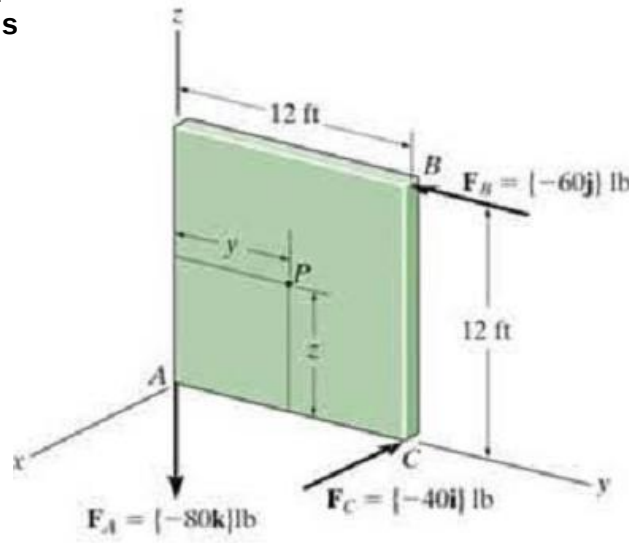
$$x\vec{i}+y\vec{j}+z\vec{k}=(2\lambda)\vec{i}+(\frac{5}{13})\vec{j}+(-10\lambda)\vec{k}$$

3- Replace the loading system by

A. An equivalent resultant force and couple.

B. A wrench specify the pitch and the axis and the point P(x, y) where its line of action intersects the plate.

Solution



E.

$$\vec{F}_A = -80\vec{k}$$
$$\vec{F}_B = -60\vec{j}$$
$$\vec{F}_C = -40\vec{i}$$
$$\vec{R} = -40\vec{i} - 60\vec{j} - 80\vec{k}$$
$$\vec{M}_0 = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$
$$M_x = 720, M_y = 0, M_z = 480$$
$$\vec{M}_0 = 720\vec{i} + 480\vec{k}$$

F.

$$|R| = 20\sqrt{26}$$
$$|M_0| = 865.33$$
$$M_{||} = \frac{\vec{R}.\vec{M}}{|R|} = \frac{67200}{20\sqrt{26}} = 623.936$$
$$P = \frac{M_{||}}{|R|} = \frac{\vec{R}.\vec{M}}{|R|^2} = \frac{623.936}{20\sqrt{26}} = 6.12$$
$$\vec{r} = \frac{\vec{R}\times\vec{M}}{|R|^2} + \lambda\vec{R} =$$
$$\frac{1}{(20\sqrt{26})^2}\begin{vmatrix} i & j & k \\ -40 & -60 & -80 \\ 720 & 0 & 480 \end{vmatrix} + \lambda(-40\vec{i} - 60\vec{j} - 80\vec{k})$$
$$= -\frac{36}{13}\vec{i} - \frac{48}{13}\vec{j} + \frac{54}{13}\vec{k} + \lambda(-40\vec{i} - 60\vec{j} - 80\vec{k}) =$$
$$x\vec{i} + y\vec{j} + z\vec{k} = (-\frac{36}{13} - 40\lambda)\vec{i} + (-\frac{48}{13} - 60\lambda)\vec{j} + (\frac{54}{13} - 80\lambda)\vec{k}$$
$$x = -\frac{36}{13} - 40\lambda, y = -\frac{48}{13} - 60\lambda, z = \frac{54}{13} - 80\lambda$$

The point p= (0, y, z)

$$0 = -\frac{36}{13} - 40\lambda \longrightarrow \lambda = -0.04, y = -1.29, z = 7.35$$

4.

Determine the resultant of the force and couple system which acts on the rectangular solid.

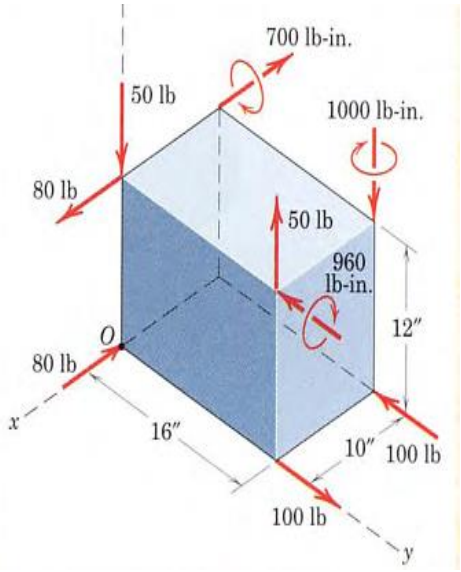
Solution. We choose point O as a convenient reference point for the initial step of reducing the given forces to a force–couple system. The resultant force is

$$\mathbf{R} = \Sigma\mathbf{F} = (80 - 80)\mathbf{i} + (100 - 100)\mathbf{j} + (50 - 50)\mathbf{k} = \mathbf{0} \text{ lb}$$

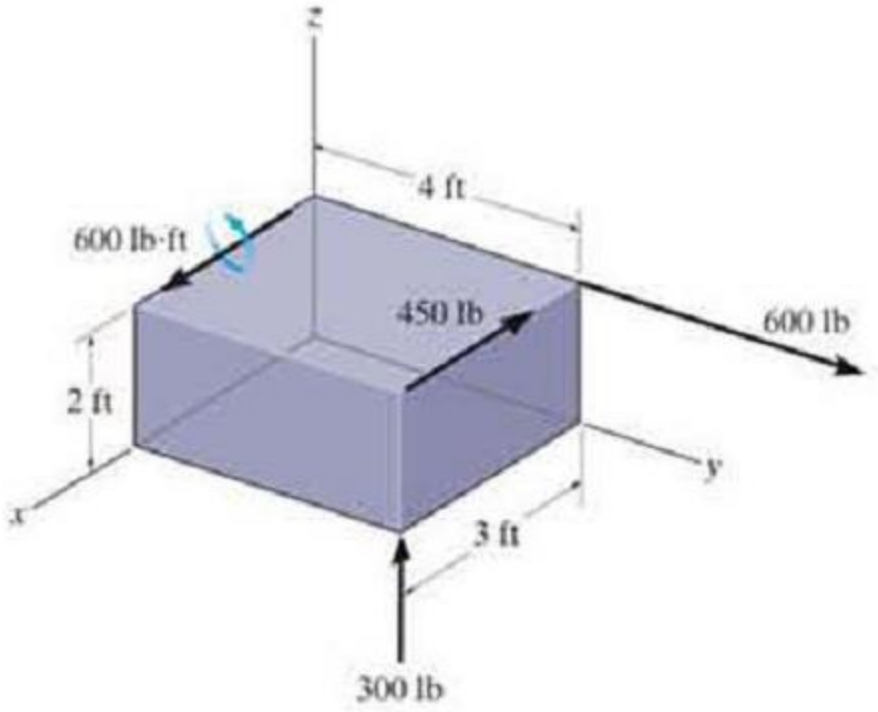
The sum of the moments about O is

$$\mathbf{M}_O = [50(16) - 700]\mathbf{i} + [80(12) - 960]\mathbf{j} + [100(10) - 1000]\mathbf{k} \text{ lb-in.}$$
$$= 100\mathbf{i} \text{ lb-in.}$$

Hence, the resultant consists of a couple, which of course may be applied at any point on the body or the body extended.

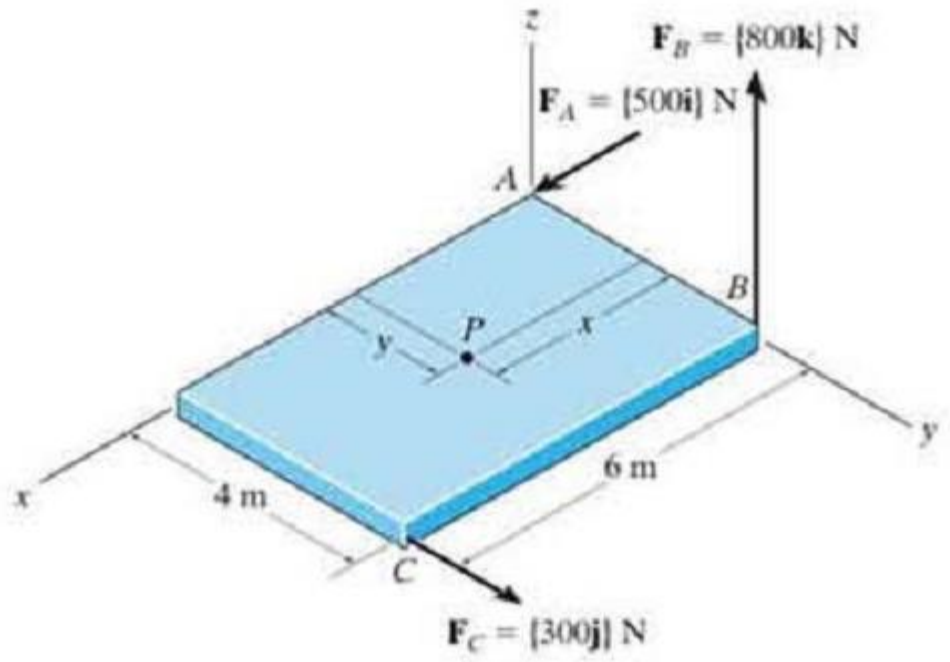


5. Replace the force and couple moment system acting on the rectangular block by a wrench. Specify the magnitude of the force and couple moment of the wrench and where its line of action intersects the x–y plane.



5.

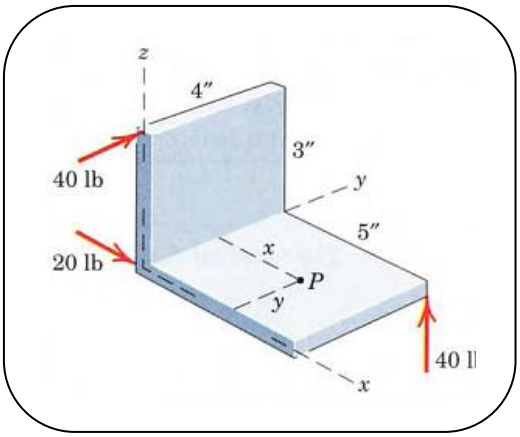
6. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



7- Replace the loading system by

A. An equivalent resultant force and couple.

B. A wrench specify the pitch and the axis and the point P(x, y) where its line of action intersects the plate.



8. Replace the two forces and single couple by an equivalent force–couple system at point A.

Ans. $\mathbf{R} = -20\mathbf{i} - 37.9\mathbf{j} + 12.65\mathbf{k}$ kN

$\mathbf{M} = 45.3\mathbf{j} + 40.9\mathbf{k}$ kN·m